Date: February _____, 2010

Title: Permutations and Combinations part 2

Objective: To find the number of ways things can be grouped when order doesn't matter.

IN:

Combination or permutation?

Picking a team of 3 people from a group of 10.

Picking a President, Vice President and a Chief Financial Officer from a group of 10.

1: Alfonso
2: Bob
3: Charlie
4: Davon
5: Emilio
6: Frank
7: George
8: Horatio

Combinations

Combinations are easy going.
Order doesn't matter.

You can mix it up and it looks the same.

Instead of medals, what if we were to give out tin cans?

How many ways can I give 3 tin cans to 8 people?

Well, in this case, the order we pick people doesn't matter.

If I give a can to Alfonso, Bob and then Charlie, it's the same as giving to Charlie, Alfonso and then Bob. Either way, they're going to be equally disappointed.

This raises and interesting point - we've got some ***redundancies*** here.
Alfonso ~ Bob ~ Charlie = Charlie ~ Bob ~ Alfonso.

For a moment, let's just figure out how many ways we can rearrange 3 people.

Well, we have 3 choices for the first person, 2 for the second, and only 1 for the last. So we have 3 * 2 * 1 ways to re-arrange 3 people.

Wait a minute… this is looking a bit like a permutation!

If you have N people and you want to know how many arrangements there are for all of them, it's just N factorial or N!

So, if we have 3 tin cans to give away, there are 3! or 6 variations for every choice we pick.

If we want to figure out how many combinations we have, we just create all the permutations and divide by all the redundancies.

In our case, we get 336 permutations (from before), and we divide by the 6 redundancies for each permutation and get 336/6 = 56.

The general formula is $$C(n,k) = \frac{P(n,k)}{k!}$$

which means “Find all the ways to pick k people from n, and divide by the k! variants”. Writing this out, we get our combination formula, or the number of ways to combine k items from a set of n:

$$C(n,k) = \frac{n!}{(n-k)!k!}$$

Combination:
Picking a team of 3 people from a group of 10.
C(10,3) = 10!/(7! * 3!) = 10 * 9 * 8 / (3 * 2 * 1) = 120.

Permutation:
Picking a President, Vice President and a CFO
from a group of 10.

P(10,3) = 10!/7! = 10 * 9 * 8 = 720.

Combination:
Choosing 3 desserts from a menu of 10.
C(10,3) = 120.

Permutation:
Listing your 3 favorite desserts, in order, from a menu of 10.
P(10,3) = 720.

Don’t memorize the formulas - it’s better to know why they work. Combinations sound simpler than permutations, and they are. You have fewer combinations than permutations.

There is a calculator shortcut

C stands for combination

$$_nC_r = \frac{n!}{(n-r)!\bullet r!}$$

Lets look at the letters in the word Awest again. If we want to use all of the letters, n would be __and r would be __.

Step 1: type the number 5
Step 2: hit the math button
Step 3: scroll to the right until you get to PRB
Step 4: Scroll down to $_nC_r$ and hit enter
Step 5: type the number 5 and hit enter

W2L

After using permutation and then combination with the word Awest, I have noticed that...

Selecting three students to attend a conference in Washington, D.C.

○ permutation ○ combination

Selecting a lead and an understudy for a school play.

○ permutation ○ combination

Assigning students to their seats on the first day of school.

○ permutation ○ combination

Evaluate: ${}_4P_2 \bullet {}_5P_3$

Choose:

○ 12
○ 60
○ 480
○ 720

The local Family Restaurant has a daily breakfast special in which the customer may choose one item from each of the following groups:

Breakfast Sandwich	Accompaniments	Juice
egg and ham egg and bacon egg and cheese	breakfast potatoes apple slices fresh fruit cup pastry	orange cranberry tomato apple grape

a.) How many different breakfast specials are possible?

b.) How many different breakfast specials without meat are possible?

a.) Basic counting principle:

Sandwiches x Accompaniments x Juice

$3 \cdot 4 \cdot 5 = 60$ breakfast choices

b.) Meatless means that under Sandwiches there will be only one choice.

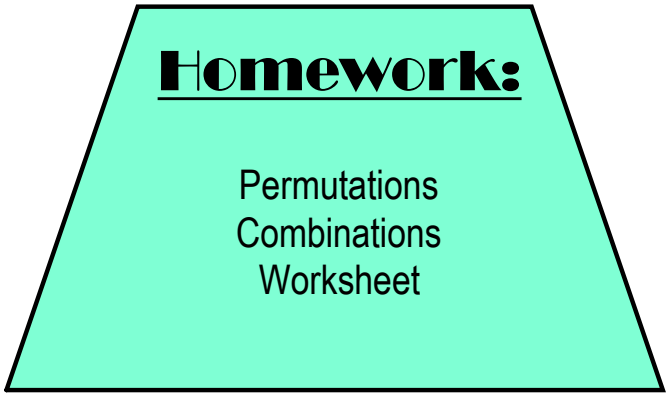
Sandwiches x Accompaniments x Juice

$1 \cdot 4 \cdot 5 = 20$ meatless breakfast choices

A coach must choose five starters from a team of 12 players. How many different ways can the coach choose the starters?

There are fourteen juniors and twenty-three seniors in the Service Club. The club is to send four representatives to the State Conference. How many different ways are there to select a group of four students to attend the conference?

Out: You are eating an ice cream cone with 6 scoops, all different flavors (yum)! How many ways can you arrange the scoops?



W2L:

I was able to clear up my confusion about ___ today.